D 12034 (Pages : 3) Name..

Reg. No.....................................

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2021

Mathematics

MTS 3C 03—MATHEMATICS – 3

(2019–2020 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

*Answer at least **eight** questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Evaluate $\int\limits_{0}^{1}\left(t\hat{i} + 3t^2\hat{j} + 4t^3\hat{k}\right)dt$.

2. The position of a moving particle is $\bar{r}(t) = t^2\hat{i} + t\hat{j} + t^3\hat{k}$. Find velocity and acceleration of the particle at $t = 2$.

3. If $z = e^{-y}\cos x$ find $\dfrac{\partial^2 z}{\partial x\,\partial y}$.

4. Find the level surface of $F(x,y,z) = x^2 + y^2 + z^2$ passing through $(1, 1, 1)$.

5. Evaluate $\oint\limits_{C} x\,dx$, where C is the circle $x = \cos t, y = \sin t, 0 \leq t \leq 2\pi$.

6. Show that $\operatorname{curl}\vec{r} = \vec{0}$.

7. State Green's theorem in the plane.

8. Evaluate $\int\limits_{0}^{3}\int\limits_{0}^{2}\int\limits_{0}^{1} xyz\,dx\,dy\,dz$.

9. Write the equation of the circle with centre $(1, 2)$ and radius 4 in the complex plane.

Turn over

10. Find the value of i^{2i}.

11. Evaluate $\oint\limits_{C} \dfrac{Z e^{z}}{(z-3)}\, dz$, where C is $|z| = 2$.

12. Evaluate $\oint\limits_{C} \dfrac{dz}{z}$, where C is $|z| = 1$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Use chain rule to find $\dfrac{dw}{dx}$ at $(0, 1, 2)$ for $w = xy + yz$; $x = \cos x,\ y \sin x,\ z = e^{x}$.

14. Find the directional derivative of $f(x, y) = \sqrt{x^{2} y + 2 y^{2} z}$ at $(-2, 2, 1)$ in the direction of the negative z-axis.

15. Find the area lying between the parabola $y = 4x - x^{2}$ and the line $y = x$ using double integrals.

16. Use polar coordinates to evaluate $\displaystyle\int\limits_{0}^{2} \int\limits_{x}^{\sqrt{8-x^{2}}} \dfrac{1}{5 + x^{2} + y^{2}}\, dy\, dx$.

17. Show that $f(z) = \left(2x^{2} + y\right) + i\left(y^{2} - x\right)$ is not analytic at any point.

18. Evaluate $\oint\limits_{C} \dfrac{5z + 7}{z^{2} + 2z - 3}\, dz$, where C is the circle $|z - 2| = 2$.

19. Evaluate $\displaystyle\int \operatorname{Re} z\, dz$ along a line segment from $z = 0$ to $z = 1 + 2i$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Let $\vec{F}(x,y,z) = z\hat{j} + z\hat{k}$ represents the flow of a liquid. Find the flux of $\vec{F}$ through the surface S given by that portion of the plane $z = 6 - 3x - 2y$ in the first octant oriented upward.

21. Use triple integrals to find the volume of the solid with in the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.

$(1 \times 11 = 11 \text{ marks})$